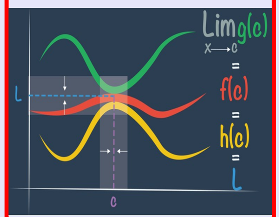
Calculus I

Lecture 44



Feb 19-8:47 AM

Anti derivative

Suppose $f'(x)$ is given, what would be $f(x)$?

ex: $f'(x) = 2x + 0 \rightarrow f(x) = \boxed{x^2} + C$

Constant

ex: $f'(x) = \cos x - 3x^2 + 0 \rightarrow f(x) = \boxed{\sin x} - \boxed{x^3} + C$

$$\boxed{f(x) = \sin x - x^3 + C}$$

ex: $f'(x) = \sec^2 x + \sin x + 8$

$f(x) = \tan x + {}^-\cos x + 8x + C$

$$\boxed{f(x) = \tan x - \cos x + 8x + C}$$

Can we find the value for the constant?

Given $f'(x) = 2x - 5$, and $f(1) = 2$

$f(x) = x^2 - 5x + C$

$f(1) = 1^2 - 5(1) + C = 2$

$1 - 5 + C = 2$

$-4 + C = 2$

$\boxed{C = 6}$

$$\boxed{f(x) = x^2 - 5x + 6}$$

May 6-8:47 AM

Recall the power rule:

$$\frac{d}{dx}\left[x^{n+1}\right] = (n+1)\,x^{n+1-1} = (n+1)\,x^{n}$$

$$\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}\right] = \frac{1}{\cancel{n+1}}\cdot\cancel{(n+1)}\cdot x^{n+\cancel{1}\cancel{-1}}$$

$$= x^{n}$$

$$\frac{d}{dx}\left[\frac{x^{n+1}}{n+1}\right] = x^{n}$$

So $f'(x) = x^{n}$

then $f(x) = \frac{1}{n+1}x^{n+1} + C$

$$f'(x) = x^{5} + 3x^{2} - 6x$$

$$f(x) = \frac{x^{6}}{6} + \frac{\cancel{3}x^{3}}{\cancel{3}} - \frac{\cancel{6}x^{2}}{\cancel{2}} + C$$

$$\boxed{f(x) = \frac{1}{6}x^{6} + x^{3} - 3x^{2} + C}$$

May 6-8:57 AM

Given $f'(x) = 5x^{4} - 3x^{2} + 4$, $f(-1) = 2$

Find $f(x)$.

$$f(x) = \frac{5x^{5}}{5} - \frac{3x^{3}}{3} + 4x + C$$

$$f(x) = x^{5} - x^{3} + 4x + C$$

$$f(-1) = (-1)^{5} - (-1)^{3} + 4(-1) + C = 2$$

$$-1 + 1 - 4 + C = 2 \rightarrow \boxed{C = 6}$$

$$\boxed{f(x) = x^{5} - x^{3} + 4x + 6}$$

May 6-9:03 AM

$f''(x) = 20x^3 + 12x^2 + 4$, $f(0)=8$, $f(1)=5$

Find $f(x)$

$f'(x) = \frac{20x^4}{4} + \frac{12x^3}{3} + 4x + C$

$f'(x) = 5x^4 + 4x^3 + 4x^1 + C$

$f(x) = \frac{5x^5}{5} + \frac{4x^4}{4} + \frac{4x^2}{2} + Cx + D$

$f(x) = x^5 + x^4 + 2x^2 + Cx + D$

$f(0) = 0^5 + 0^4 + 2(0)^2 + C(0) + D = 8$

$\boxed{D=8}$

$f(x) = x^5 + x^4 + 2x^2 + Cx + 8$

$f(1) = 1^5 + 1^4 + 2(1)^2 + C(1) + 8 = 5$

$1 + 1 + 2 + C + 8 = 5$

$\boxed{C=-7}$

$\boxed{f(x) = x^5 + x^4 + 2x^2 - 7x + 8}$

May 6-9:10 AM

$f''(x) = 2 + \cos x$, $f(0)=-1$, $f(\frac{\pi}{2})=0$

$f'(x) = 2x + \sin x + C$

$f(x) = x^2 - \cos x + Cx + D$

$f(0) = \cancel{0^2}^{\,0} - \cancel{\cos 0}^{\,1} + \cancel{C(0)}^{\,0} + D = -1$

$-1 + D = -1 \rightarrow \boxed{D=0}$

$f(x) = x^2 - \cos x + Cx$

$f(\frac{\pi}{2}) = (\frac{\pi}{2})^2 - \cancel{\cos\frac{\pi}{2}}^{\,0} + C(\frac{\pi}{2}) = 0$

$\frac{\pi^2}{4} + \frac{\pi}{2}C = 0$

Multiply by 4

$\pi^2 + 2\pi C = 0$

$2\pi C = -\pi^2$

$C = \frac{-\pi^2}{2\pi}$

$\boxed{C = -\frac{\pi}{2}}$

$\boxed{f(x) = x^2 - \cos x - \frac{\pi}{2}x}$

May 6-9:17 AM

$f'(x) = \sqrt{x}(6+5x)$, $f(1) = 10$, find $f(x)$.

$f'(x) = 6\sqrt{x} + 5x\sqrt{x}$

$f'(x) = 6x^{\frac{1}{2}} + 5x^{1} \cdot x^{\frac{1}{2}}$

$f'(x) = 6x^{\frac{1}{2}} + 5x^{\frac{3}{2}}$

$f(x) = \dfrac{6x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \dfrac{5x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + C$

$f(x) = \dfrac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \dfrac{5x^{\frac{5}{2}}}{5/2} + C$

$f(x) = 4x\sqrt{x} + 2x^2\sqrt{x} + C$

$f(1) = 4 \cdot 1 \cdot \sqrt{1} + 2 \cdot 1^2 \cdot \sqrt{1} + C = 10$

$4 + 2 + C = 10$ $\boxed{C = 4}$

$\boxed{f(x) = 4x\sqrt{x} + 2x^2\sqrt{x} + 4}$

May 6-9:23 AM

$f''(x) = \sin x + \cos x$

$f(0) = 3$

$f'(0) = 4$

Find $f(x)$

$f'(x) = -\cos x + \sin x + C$

$f'(0) = -\cancel{\cos 0}^{\,1} + \cancel{\sin 0}^{\,0} + C = 4$

$-1 + C = 4$

$\boxed{C = 5}$

$f'(x) = -\cos x + \sin x + 5$

$f(x) = -\sin x - \cos x + 5x + C$

$f(0) = -\cancel{\sin 0}^{\,0} - \cancel{\cos 0}^{\,1} + \cancel{5(0)}^{\,0} + C = 3$

$\boxed{C = 4}$

$\boxed{f(x) = -\sin x - \cos x + 5x + 4}$

May 6-9:31 AM

Class QZ 20

Given $f(x) = \dfrac{x}{x-4}$

1) Find $f'(x)$, give x-values where $f'(x)=0$ or undefined

$f'(x) = \dfrac{1(x-4) - x \cdot 1}{(x-4)^2} = \dfrac{-4}{(x-4)^2}$

$f'(x) \neq 0$

$f'(x)$ undefined at $x=4$

2) Find $f''(x)$, give x-values where $f''(x)=0$ or undefined.

$f'(x) = -4(x-4)^{-2}$

$f''(x) = 8(x-4)^{-3} = \dfrac{8}{(x-4)^3}$

$f''(x) \neq 0$

$f''(x)$ undefined at $x=4$

3) Complete the Sign chart.

x	$-\infty$		4		∞
$f'(x)$		$-$	ϕ	$-$	
$f''(x)$		$-$	ϕ	$+$	
$f(x)$		$\searrow$		$\searrow$	

May 6-9:41 AM